Published by Institute of Physics Publishing for SISSA

Received: *April 28, 2005*
Revised: *June 21, 2005*
Accepted: *June 22, 2005*
Published: *July 25, 2005*

Moduli instability in warped compactifications of the type-IIB supergravity

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Abstract: We show that the conifold and deformed-conifold warped compactifications of the ten-dimensional type-IIB supergravity, including the Klebanov-Strassler solution, are dynamically unstable in the moduli sector representing the scale of a Calabi-Yau space, although it can be practically stable for a quite long time in a region with a large warp factor. This instability is associated with complete supersymmetry breaking except for a special case and produces significant time-dependence in the structure of the four-dimensional base spacetime as well as of the internal space.

Keywords: Supersymmetry Breaking, Supergravity Models.

JHEP07(2005)061

Contents

1. Introduction

With the practical confirmation of the inflationary universe scenario of the early universe and the discovery of the accelerating expansion of the present universe [1 − 5], it is now the most challenging problem to construct a consistent cosmological model that explains these observational facts, on the basis of supergravity and string theory, which are the only viable unified fundamental theories at present. The main obstacle to this problem is the fact that these theories require the spacetime to be higher dimensional; in order to obtain a four-dimensional universe at low energies, we have to find a natural way to conceal extra dimensions, which is usually called a compactification. This compactification gives rise to various new problems. One of the most serious problems is the moduli stabilisation [6]. Another is the no-go theorem against accelerating expansion of the universe in simple Kaluza-Klein-type or stationary warped compactification with a smooth compact internal space [7].

Recently, a new progress in resolving these problems has been made by Kachru et al [8, 9]. Utilising a conifold-type flux compactification of the IIB supergravity that realises the stabilisation of all complex moduli [8, 10], they proposed a model in which all moduli are potentially stabilised and an accelerating expansion of the universe is realised for a sufficiently long time. There is, however, one subtle weak point in their model. It is the stability of the moduli degree of freedom corresponding to the scale of the Calabi-Yau internal space [11]. They argued that this degree of freedom would be stabilised by quantum nonperturbative effects [12 − 14]. However, their argument is based on a four-dimensional effective theory that does not take into account the warping and assumes the supersymmetric background.

The main purpose of the present paper is to analyse the stability of this moduli degree of freedom in the classical framework of the full ten-dimensional IIB supergravity, taking account of the warped structure. We show that many of well-known supersymmetric compactifications of the type-IIB supergravity by a conifold [15 − 17], a resolved conifold [18] or a deformed conifold [10, 19, 20] with 5-form flux and 3-form flux are limits of dynamical solutions with four extra parameters. This instability occurs in the warp factor in the form $h = h_1(y) + a_\mu x^\mu + b$, where x^μ and y^p are the coordinates of the four-dimensional base spacetime and the Calabi-Yau internal space, respectively, and for $a_\mu = b = 0$, the solution reduces to the original supersymmetric one.

In the special case in which there is no 3-form flux, the corresponding solution is identical to the one recently found by Gibbons, Lü and Pope [21] as a higher-dimensional analogue of the four-dimensional Kastor-Traschen solution [22]. In the case of a deformed conifold with 5-form and 3-form fluxes, the corresponding solution provides a time-dependent extension of the Klebanov-Strassler solution [10]. We show that supersymmetry is fully broken, except in the case the solution has a null Killing, for the Gibbons-Lü-Pope solution.

The present paper is organised as follows. First, in section 2, after making clear ansatz to be imposed on various fields and the spacetime metric of the IIB supergravity to find solutions, we show that all field equations including the Einstein equations can be reduced under the ansatz to a simple set of equations on the Calabi-Yau manifold for the 2-form potential B_2 and the warp factor h. We also show that h is restricted to the form given above in general. Then, in section 3, we apply this formulation to various compactification models and derive explicit expressions for the solutions. In section 4 we study the supersymmetry breaking for the Gibbons-Lü-Pope solution and discuss its implication. Finally, section 5 is devoted to summary and discussion.

2. Formulation

2.1 Ansatz

In the present paper, we look for solutions whose spacetime metric has the form

$$ds^2 = A^2(x,y)\, ds^2(\mathrm{X}_4) + B^2(x,y)\, ds^2(\mathrm{Y}_6)\,, \tag{2.1}$$

where $ds^2(\mathrm{X}_4)$ denotes the four-dimensional metric depending only on the coordinates x^μ of X_4, and $ds^2(\mathrm{Y}_6)$ denotes the six-dimensional metric depending only on the coordinates y^p

of Y_6. Hence, the dynamics is essentially limited to the scale factors A and B. Concerning the other fields, we adopt the following assumptions

$$\tau \equiv C_0 + i\, e^{-\Phi} = i g_s^{-1}\,, \tag{2.2a}$$

$$G_3 \equiv i g_s^{-1} H_3 - F_3 = \frac{1}{3!} G_{pqr}\, dy^p \wedge dy^q \wedge dy^r\,, \tag{2.2b}$$

$$*_{\mathrm{Y}}\, G_3 = \epsilon\, i\, G_3 \qquad (\epsilon = \pm 1)\,, \tag{2.2c}$$

$$d * (B_2 \wedge F_3) = 0\,, \tag{2.2d}$$

$$\tilde{F}_5 = A^4 B^{-4} (1 \pm *) V_p\, dy^p \wedge \Omega(\mathrm{X}_4) = A^4 B^{-4} V \wedge \Omega(\mathrm{X}_4) \mp *_{\mathrm{Y}}\, V\,, \tag{2.2e}$$

where g_s is a constant representing the string coupling constant, and $*$ and $*_{\mathrm{Y}}$ are the Hodge duals with respect to the ten-dimensional metric ds^2 and the six-dimensional metric $ds^2(\mathrm{Y}_6)$, respectively.

2.2 Reduction of the gauge field equations

Under the assumptions given above, we first reduce the field equations other than the Einstein equations to a simple set of equations.

The gauge field equations are given by [23, 24]

$$\Box\tau + i\frac{(\nabla\tau)^2}{\tau_2} = -\frac{i}{2} G_3 \cdot G_3\,, \tag{2.3a}$$

$$dG_3 = 0\,, \tag{2.3b}$$

$$\nabla \cdot G_3 \equiv *d * G_3 = i G_3 \cdot \tilde{F}_5\,, \tag{2.3c}$$

$$\tilde{F}_5 = dC_4 + B_2 \wedge F_3\,, \tag{2.3d}$$

$$*\tilde{F}_5 = \pm \tilde{F}_5\,. \tag{2.3e}$$

In the present paper, we define the inner product of a p-form ω_p and a q-form χ_q ($p \le q$) as

$$(\omega_p \cdot \chi_q)_{\mu_1\cdots\mu_{q-p}} := \frac{1}{p!} \omega^{\nu_1\cdots\nu_p} \chi_{\nu_1\cdots\nu_p \mu_1\cdots\mu_{q-p}}\,. \tag{2.4}$$

Note that the first equation is automatically satisfied under our ansatz.

Under the assumption (2.2b), the equation (2.3b) implies that G_3 is a closed form depending only on the coordinates of Y_6. Then, from the self-duality requirement for G_3, (2.2c), it follows that G_3 can be expressed in terms of a 2-form α_2 on Y_6 satisfying $d *_{\mathrm{Y}} d\alpha_2 = 0$ as

$$G_3 = i\, d\alpha_2 + \epsilon *_{\mathrm{Y}} d\alpha_2\,. \tag{2.5}$$

Therefore, B_2 and F_3 are expressed as

$$B_2 = g_s\, \alpha_2\,, \qquad F_3 = -\epsilon *_{\mathrm{Y}} d\alpha_2\,. \tag{2.6}$$

Next we consider the 5-form $\tilde{F}_5$. First, note that for any q-form ω_q on Y_6, the Hodge dual operators $*$ and $*_{\mathrm{Y}}$ are related by

$$*\omega_q = A^4 B^{6-2q}\, \Omega(\mathrm{X}_4) \wedge *_{\mathrm{Y}}\, \omega_q\,, \tag{2.7a}$$
$$*\, [\Omega(\mathrm{X}_4) \wedge \omega_q] = -A^{-4} B^{6-2q}\ *_{\mathrm{Y}}\, \omega_q\,. \tag{2.7b}$$

Further, the operator $*_{\mathrm{Y}}$ satisfies the relations

$$*_{\mathrm{Y}}(\alpha_2 \cdot d\alpha_2)_Y = -\alpha_2 \wedge *_{\mathrm{Y}} d\alpha_2 \qquad *_{\mathrm{Y}} *_{\mathrm{Y}}\, \omega_q = (-1)^q\, \omega_q\,, \tag{2.8}$$

where $(\omega \cdot \chi)_Y$ denote the inner product of forms ω and χ on Y_6 with respect to the metric $ds^2(\mathrm{Y}_6)$. From these relations, we get

$$*(B_2 \wedge F_3) = -g_s \epsilon A^4 B^{-4}\, \Omega(\mathrm{X}_4) \wedge \beta_1\,, \tag{2.9}$$

where β_1 is the 1-form defined as $\beta_1 = (\alpha_2 \cdot d\alpha_2)_Y$. Hence, the equation (2.2d) gives the condition

$$d_y(A^4 B^{-4} \beta_1) = 0\,. \tag{2.10}$$

Under this condition, the assumption (2.2e) on $\tilde{F}_5$ can be rewritten as

$$(1 \pm *) h^{-2} V \wedge \Omega(\mathrm{X}_4) = d\tilde{C}_4 \mp \epsilon g_s A^4 B^{-4} (1 \pm *) \beta_1 \wedge \Omega(\mathrm{X}_4)\,, \tag{2.11}$$

where $\tilde{C}_4$ is some 4-form. From this, it follows that V and $\tilde{C}_4$ can be written in terms of β_1 and some 1-form γ_1 as

$$V = \gamma_1 \mp \epsilon g_s\, \beta_1\,, \tag{2.12a}$$
$$d\tilde{C}_4 = A^4 B^{-4} (1 \pm *) \gamma_1 \wedge \Omega(\mathrm{X}_4) = A^4 B^{-4}\, \gamma_1 \wedge \Omega(\mathrm{X}_4) \mp *_{\mathrm{Y}} \gamma_1\,. \tag{2.12b}$$

From this and the self duality of $\tilde{F}_5$, γ_1 is required to satisfy

$$\partial_x \gamma_1 = 0\,, \qquad d_y(A^4 B^{-4}\, \gamma_1) = 0\,, \qquad \hat{D} \cdot \gamma_1 = 0\,, \tag{2.13}$$

where $\hat{D}_p$ is the covariant derivative with respect to the metric $ds^2(\mathrm{Y}_6)$. In particular, γ_1 is a 1-form on Y_6 independent of x.

Finally, utilising the relations

$$G_3 \cdot \tilde{F}_5 = \pm i\epsilon\, B^{-6} (V \cdot G_3)_Y\,, \tag{2.14a}$$
$$*d * G_3 = A^{-4} B^{-2} \left(d_y(A^4) \cdot G_3 \right)_Y\,, \tag{2.14b}$$

the remaining field equation (2.3c) can be replaced by the simple equation

$$[V \pm \epsilon A^{-4} B^4 d_y(A^4)] \cdot G_3 = 0\,. \tag{2.15}$$

Since G_3 is self-dual on Y_6, if $G_3 \neq 0$, this equation is equivalent to

$$V = \mp \epsilon A^{-4} B^4 d_y(A^4)\,. \tag{2.16}$$

Since β_1 and γ_1 are 1-forms on Y_6, this equation leads to

$$\partial_\mu(A^{-1}B^4\partial_p A) = 0\,. \tag{2.17}$$

Further, under (2.10), the equations (2.13) are equivalent to

$$\hat{D}\cdot[A^{-4}B^4\hat{D}(A^4)] = g_s(d\alpha_2\cdot d\alpha_2)_Y\,. \tag{2.18}$$

To summarise, if we find a 2-form α_2 on Y_6 and functions $A(x,y)$ and $B(x,y)$ satisfying $d*_{\mathrm{Y}}d\alpha_2 = 0$, (2.10), (2.17) and (2.18), then B_2 and F_3 given by (2.6) and $\tilde{F}_5$ given by

$$\tilde{F}_5 = \mp\epsilon(1\pm *)d(A^4)\wedge\Omega(\mathrm{X}_4) \tag{2.19}$$

satisfy the field equations other than the Einstein equations. Note that this yields the most general solution under our ansatz in the case $G_3\neq 0$, while in the case $G_3 = 0$ it may be a special solution.

2.3 The Einstein equations

In order to complete the system of equations, we must also consider the Einstein equations [23, 24]

$$R_{MN} = \frac{g_s}{4}\left[\mathrm{Re}(G_{MPQ}G^{*\,PQ}_N) - \frac{1}{2}\,G_3\cdot G_3^*\,g_{MN}\right] + \frac{1}{96}\,\tilde{F}_{MP_1\cdots P_4}\,\tilde{F}_N{}^{P_1\cdots P_4}\,. \tag{2.20}$$

Under our ansatz, these equations become

$$R_{\mu\nu} = -\frac{1}{4}S(x,y)\,g_{\mu\nu}\,, \tag{2.21a}$$
$$R_{\mu p} = 0\,, \tag{2.21b}$$
$$R_{pq} = \frac{1}{4}S(x,y)\,g_{pq} - \frac{V_pV_q}{2h^2}\,, \tag{2.21c}$$

where $S(x,y)$ is

$$S(x,y) = g_s\,B^{-6}\,(d\alpha_2\cdot d\alpha_2)_Y + B^{-10}\,(V\cdot V)_Y\,. \tag{2.22}$$

First, if we introduce new set of variables $h(x,y)$ and $a(x,y)$ by $B = h^{1/4}$ and $A = ah^{-1/4}$, the equation (2.21b) can be written

$$R_{\mu p} = -\frac{1}{2h}\partial_\mu\partial_p h + \frac{2}{ah}\partial_p a\partial_\mu h - \frac{3}{a}\partial_\mu\partial_p a + \frac{3}{a^2}\partial_\mu a\partial_p a = 0\,. \tag{2.23}$$

Further, (2.17) reads

$$-\frac{1}{h}\partial_\mu\partial_p h + \frac{4}{ah}\partial_p a\partial_\mu h + \frac{4}{a}\partial_\mu\partial_p a - \frac{4}{a^2}\partial_\mu a\partial_p a = 0\,. \tag{2.24}$$

From these two equations, we obtain $\partial_\mu\partial_p\ln a = 0$. This implies that a can be written $a = a_0(x)a_1(y)$. Therefore, we can set $a = 1$ by redefining $ds^2(\mathrm{X}_4), ds^2(\mathrm{Y}_6)$ and h. Then, these equations reduce to $\partial_\mu\partial_p h = 0$. Hence, h can be expressed as $h = h_0(x) + h_1(y)$. Further, V can be written as $V = \pm\epsilon\,d_y h$, and (2.18) reads $\triangle_Y h_1 = -g_s(d\alpha_2\cdot d\alpha_2)_Y$.

Next, taking account of these results, the equation (2.21a) yields

$$R_{\mu\nu}(\mathrm{X}_4) - h^{-1}D_\mu D_\nu h_0 = -\frac{1}{4}h^{-1}\triangle_X h_0\, g_{\mu\nu}(\mathrm{X}_4)\,, \tag{2.25a}$$
$$R(\mathrm{X}_4) = 0\,, \tag{2.25b}$$

where $\triangle_X$ is the laplacian with respect to the metric $ds^2(\mathrm{X}_4)$. If we require that $d_y h \neq 0$, these equations can be reduced to

$$R_{\mu\nu}(\mathrm{X}_4) = 0\,, \qquad D_\mu D_\nu h_0 = \frac{1}{4}\triangle_X h_0\, g_{\mu\nu}(\mathrm{X}_4)\,. \tag{2.26}$$

Finally, taking account of the Poisson equation for h and the expression of V in terms of h again, (2.21c) reduces to

$$R_{pq}(\mathrm{Y}_6) = \frac{1}{6}R(\mathrm{Y}_6)g_{pq}(\mathrm{Y}_6)\,, \qquad R(\mathrm{Y}_6) = \frac{3}{2}\triangle_X h_0\,. \tag{2.27}$$

These equations are equivalent

$$R_{pq}(\mathrm{Y}_6) = \lambda g_{pq}(\mathrm{Y}_6)\,, \qquad \triangle_X h_0 = 4\lambda\,, \tag{2.28}$$

where λ is a constant.

To summarise, for any h of the form

$$h = h_0(x) + h_1(y) \tag{2.29}$$

and 2-form B_2 on Y_6 satisfying

$$d *_Y dB_2 = 0\,, \qquad d_y[h^{-2}(B_2 \cdot dB_2)_Y] = 0\,, \tag{2.30a}$$
$$\triangle_Y h_1 = -g_s^{-1}(dB_2 \cdot dB_2)_Y\,, \tag{2.30b}$$
$$D_\mu D_\nu h_0 = \lambda g_{\mu\nu}(X)\,, \tag{2.30c}$$

the metric

$$ds^2(M_{10}) = h^{-1/2}ds^2(\mathrm{X}_4) + h^{1/2}ds^2(\mathrm{Y}_6) \tag{2.31}$$

with

$$R_{\mu\nu}(\mathrm{X}_4) = 0\,, \tag{2.32a}$$
$$R_{pq}(\mathrm{Y}_6) = \lambda g_{pq}(\mathrm{Y}_6)\,, \tag{2.32b}$$

and the gauge fields given by

$$H_3 = dB_2\,, \qquad F_3 = -\epsilon g_s^{-1} *_Y H_3\,, \tag{2.33a}$$
$$\tilde{F}_5 = \mp\epsilon(1 \pm *)d(h^{-1}) \wedge \Omega(X^4) \tag{2.33b}$$

yield a solution to the type-IIB supergravity. Under our ansatz, this is the most general solution in the case $G_3 \neq 0$ and $d_y h \neq 0$, while otherwise it may be a special solution.

Here, note that the conditions (2.30c) and (2.32a) strongly restrict the metric $ds^2(\mathrm{X}_4)$. In fact, as is shown in appendix A, X_4 is required to be locally flat, irrespective of the value of λ, if $D_\mu h_0 \neq 0$ and $(Dh_0)^2 \neq 0$. In this case, the general solution for h_0 is given by

$$h_0 = \frac{\lambda}{2} x^\mu x_\mu + a_\mu x^\mu + b \tag{2.34}$$

in the Minkowki coordinates, where a_μ and b are constants satisfying the condition $a\cdot a \neq 0$. On the other hand, if $D_\mu h_0 \neq 0$ and $(Dh_0)^2 = 0$, there exists a solution only when $\lambda = 0$. The four-dimensional metric is restricted to the form

$$ds^2(\mathrm{X}_4) = \eta_{\mu\nu} dx^\mu dx^\nu + f(x, y, t - z)(dt - dz)^2 \,, \tag{2.35}$$

where $f(x, y, t - z)$ is an arbitrary solution to

$$(\partial_x^2 + \partial_y^2) f = 0 \,. \tag{2.36}$$

h_0 is expressed in these coordinates as

$$h_0 = a(t - z) + b \,, \tag{2.37}$$

where a and b are arbitrary constants. Note that when f is linear with respect to x and y, $ds^2(\mathrm{X}_4)$ is really a flat metric and f can be set to zero by a coordinate transformation. In this case, h_0 is given by (2.34) with $\lambda = 0$ and $a \cdot a = 0$.

Finally, note that in the case $G_3 = 0$, the equation (2.30a) becomes trivial, and (2.30c) reduces to $\triangle_Y h_1 = 0$. In the special case of $\lambda = 0$, the corresponding dynamical solution with (2.34) is identical to the solution found by Gibbons, Lü and Pope [21].

3. Application to the conifold-type compactifications

In this section, we apply the general formulation developed in the previous section to the flux compactification on conifold-type Calabi-Yau spaces in order to find time-dependent generalisations of the Klebanov-Strassler solutions [10].

In all cases, we look for solutions whose ten-dimensional metrics have the form

$$ds^2 = h^{-1/2}(x, r) ds^2(E^{3,1}) + h^{1/2}(x, r) ds^2(\mathrm{Y}_6) \,, \tag{3.1}$$

where $\mathrm{X}_4 = E^{3,1}$ is the four-dimensional Minkowski spacetime, Y_6 is a six-dimensional Calabi-Yau space, and r is a radial coordinate of Y_6. Further, we only consider Calabi-Yau spaces whose level surface with respect to r approaches the Einstein space T^{11} at large r (up to a scale factor), whose metric is given by [25]

$$ds^2(\mathrm{T}^{11}) = \frac{1}{9}\left(d\psi + \sum_{i=1}^{2} \cos\theta_i \, d\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i \, d\phi_i^2\right) , \tag{3.2}$$

where the range of the angular coordinates θ_i, ϕ_i and ψ are $0 \leq \theta_i < \pi$, $0 \leq \phi_i < 2\pi$ and $0 \leq \psi < 4\pi$, respectively.

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Throughout this section, we use the following orthogonal basis [10, 19]:

$$
\begin{aligned}
g^1 &= \frac{1}{\sqrt{2}}(e^1 - e^3)\,, \qquad g^2 = \frac{1}{\sqrt{2}}(e^2 - e^4)\,, \qquad g^3 = \frac{1}{\sqrt{2}}(e^1 + e^3)\,, \\
g^4 &= \frac{1}{\sqrt{2}}(e^2 + e^4)\,, \qquad g^5 = e^5\,,
\end{aligned} \tag{3.3}
$$

where

$$
\begin{aligned}
&e^1 \equiv -\sin\theta_1\, d\phi_1\,, \qquad e^2 \equiv d\theta_1\,, \qquad e^3 \equiv \cos\psi\, \sin\theta_2\, d\phi_2 - \sin\psi\, d\theta_2\,, \\
&e^4 \equiv \sin\psi\, \sin\theta_2\, d\phi_2 + \cos\psi\, d\theta_2\,, \qquad e^5 \equiv d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2\,.
\end{aligned} \tag{3.4}
$$

The line element of T^{11} is expressed in terms of this basis as

$$
ds^2(\mathrm{T}^{11}) = \frac{1}{9}(g^5)^2 + \frac{1}{6}\sum_{i=1}^{4}(g^i)^2\,. \tag{3.5}
$$

3.1 Time-dependent conifold solution

First, we consider the case in which Y_6 is a simple conifold over T^{11}:

$$
ds^2(\mathrm{Y}_6) = dr^2 + r^2 ds^2(\mathrm{T}^{11})\,. \tag{3.6}
$$

Let B_2 be a 2-form on Y_6 of the form [10]

$$
B_2 = g_s\, f(r)(g^1\wedge g^2 + g^3\wedge g^4) = g_s\, f(r)\left[\Omega(\mathrm{S}_1^2) - \Omega(\mathrm{S}_2^2)\right], \tag{3.7}
$$

where

$$
\Omega(\mathrm{S}_1^2) = \sin\theta_1\, d\theta_1\wedge d\phi_1\,, \qquad \Omega(\mathrm{S}_2^2) = \sin\theta_2\, d\theta_2\wedge d\phi_2\,. \tag{3.8}
$$

Then, from (2.33a), H_3 and F_3 are given by

$$
H_3 = dB_2 = g_s\, f'(r)dr\wedge\left[\Omega(\mathrm{S}_1^2) - \Omega(\mathrm{S}_2^2)\right], \tag{3.9a}
$$

$$
F_3 = -\epsilon g_s^{-1} *_{\mathrm{Y}} H_3 = -\epsilon\frac{1}{3}r f'(r)d\psi\wedge\left[\Omega(\mathrm{S}_1^2) - \Omega(\mathrm{S}_2^2)\right]. \tag{3.9b}
$$

Hence, the first equation of (2.30a) gives $rf' = M = \text{const}$:

$$
f(r) = M\ln\left(\frac{r}{r_0}\right), \tag{3.10}
$$

where r_0 is a constant. Further, the second equation of (2.30a) is trivially satisfied, because $(B_2\cdot H_3)_Y$ is a constant multiple of $ff'dr$ and h depends only on r for fixed x-coordinates.

The remaining non-trivial equation (2.30b) reduces to

$$
\hat{\triangle}_{\mathrm{Y}_6} h = \frac{1}{r^5}(r^5 h')' = -g_s^{-1}(H_3\cdot H_3)_Y = -\frac{72 g_s M^2}{r^6}\,. \tag{3.11}
$$

Thus, taking account of (2.29) and (2.34), the general solution for h under our ansatz is given by

$$
h(x,r) = h_0(x) + \frac{36 g_s\, M^2}{r^4}\left[\ln\left(\frac{r}{r_0}\right) + \frac{1}{4}\right] + \frac{C}{r^4}\,, \tag{3.12}
$$

where C is a constant, and $h_0(x)$ is a linear function of x^μ.

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Now, let us briefly discuss some characteristic features of this solution (cf. ref. [21]). First, in the region where $r \gg |C|, g_s M^2 |\log(r/r_0) + 1/4|$, the corresponding spacetime metric depends only on $h_0(x)$, and there appears curvature singularity at the hypersurface where $h_0(x)$ vanishes, if $Dh_0 \neq 0$. For example, for $h_0(t) = -pt + q$ $(p > 0)$, the four-dimensional metric $h^{-1/2} ds^2(E^{3,1})$ represents an expanding universe for $t < q/p$, which ends at the big-lip singularity at $t = -q/p$. This expansion is associated with contraction of the internal space. Hence, if we follow the standard prescription in which the effective scale factor of the four-dimensional universe is given by $AB^3 = h^{1/2}$, this solution is interpreted as representing a contracting universe. The converse situation arises for $p < 0$. However, this interpretation based on the effective action may not be valid in the case in which moduli are not stabilised, because changes in moduli produce changes in fundamental coupling constants, which affect the spectra of atoms for example. A correct physical interpretation should be obtained only by taking account of such effects on observations.

In contrast to this large r region, the time dependence of h in the small r region is negligible compared with the terms produced by flux, provided that $h > 0$. Hence, in this region, the scale modulus is practically stabilised for a long time. This feature may be used as a moduli stabilisation mechanism in the context of the braneworld model [26, 27], in particular for a similar solution in the deformed conifold compactification discussed later.

3.2 Resolved-conifold compactification

Next, let us consider time-dependent solutions for compactification on the resolved conifold, whose metric is given by [18, 28, 29]

$$ds^2(\mathrm{Y}_6) = \kappa^{-1}(r)dr^2 + \frac{1}{9}\kappa(r)r^2\,(e^5)^2 + \frac{1}{6}r^2 ds^2(\mathrm{S}_1^2) + \frac{1}{6}(r^2 + 6a^2)ds^2(\mathrm{S}_2^2)\,, \tag{3.13}$$

where

$$\kappa(r) = \frac{r^2 + 9a^2}{r^2 + 6a^2}\,, \tag{3.14}$$

and $ds^2(\mathrm{S}_1^2)$ and $ds^2(\mathrm{S}_2^2)$ denote the line elements of the spheres S_1^2 and S_2^2, respectively.

For this internal space, the choice of B_2 of the form [18, 29]

$$B_2 = g_s\left[f_1(r)\Omega(\mathrm{S}_1^2) + f_2(r)\Omega(\mathrm{S}_2^2)\right] \tag{3.15}$$

yields

$$H_3 = dB_2 = g_s\, dr \wedge \left[f_1'(r)\Omega(\mathrm{S}_1^2) + f_2'(r)\Omega(\mathrm{S}_2^2)\right], \tag{3.16a}$$

$$F_3 = \epsilon\frac{1}{3}g^5 \wedge \left[\frac{r^2 + 6a^2}{r} f_1'(r)\Omega(\mathrm{S}_2^2) + \frac{r^3}{r^2 + 6a^2} f_2'(r)\Omega(\mathrm{S}_1^2)\right], \tag{3.16b}$$

where the prime denotes differentiation with respect to r. From the first equation of (2.30a), $dF_3 = 0$, the functions $f_1(r)$ and $f_2(r)$ obey

$$f_1'(r) = \frac{Mr}{r^2 + 9a^2}\,, \qquad f_2'(r) = -\frac{M(r^2 + 6a^2)^2}{r^3(r^2 + 9a^2)}\,. \tag{3.17}$$

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The general solution to these equations are

$$f_1(r) = C_1 + \frac{1}{2}\ln(r^2+9a^2)\,, \tag{3.18a}$$

$$f_2(r) = C_2 + \frac{2a^2}{r^2} - \frac{1}{18}\ln\left[r^{16}(r^2+9a^2)\right]\,, \tag{3.18b}$$

where C_1 and C_2 are constants. The equation (2.30b) now reads

$$\left[r^3(r^2+9a^2)h'\right]' = -36 g_s M^2 \frac{r^4+(r^2+6a^2)^2}{r^3(r^2+9a^2)}\,, \tag{3.19}$$

where we have used

$$\beta_1 = 36 g_s \frac{dr}{r^3(r^2+9a^2)}(f_1 - f_2)\,, \tag{3.20}$$

$$H_3 \cdot H_3 = 36 g_s^2 M^2 \frac{r^4+(r^2+6a^2)^2}{r^6(r^2+6a^2)(r^2+9a^2)}\,. \tag{3.21}$$

The second of (2.30a) is again trivially satisfied. Therefore, we obtain

$$h' = \frac{4 g_s M^2}{r^3(r^2+9a^2)}\left(\frac{18a^2}{r^2} - \ln\left[r^8(r^2+9a^2)^5\right]\right) + \frac{C}{r^3(r^2+9a^2)}\,, \tag{3.22}$$

where C is a constant.

For large r $(r \gg 3a)$, we reproduce the solution (3.12) after integrating (3.22) over r, taking account of (2.29) and (2.34). On the other hand, for small r $(r \ll 3a)$, the solution is approximated by [18]

$$h(x,r) = h_0(x) - \frac{C}{18a^2r^2} - \frac{2g_sM^2}{r^4}\,, \tag{3.23}$$

where $h_0(x)$ is a linear function of x.

3.3 Deformed-conifold compactification

Finally, we show that the deformed-conifold solution also has a dynamical generalisation.

The deformed-conifold metric can be written as [10, 19]

$$\begin{aligned} ds_6^2(\mathrm{Y}_6) = {} & \frac{1}{2}\sigma^2 K(\tau)\left[\frac{1}{3K^3(\tau)}\{d\tau^2 + (g^5)^2\} + \sinh^2\left(\frac{\tau}{2}\right)\{(g^1)^2+(g^2)^2\} + \right. \\ & \left. + \cosh^2\left(\frac{\tau}{2}\right)\{(g^3)^2+(g^4)^2\}\right]\,, \end{aligned} \tag{3.24}$$

where σ is a constant, and

$$K(\tau) = \frac{[\sinh(2\tau) - 2\tau]^{1/3}}{2^{1/3}\sinh(\tau)}\,. \tag{3.25}$$

The radial coordinate r is related to τ by

$$r = 6^{-1/2}\,\sigma \int_0^\tau \frac{du}{K(u)}\,. \tag{3.26}$$

For this Calabi-Yau space, let us assume that B_2 takes the form [10]

$$B_2 = g_s \left[k_1(\tau) g^1 \wedge g^2 + k_2(\tau) g^3 \wedge g^4 \right] . \tag{3.27}$$

Then, from (2.33a), H_3 and F_3 are expressed as

$$\begin{aligned} H_3 = dB_2 = g_s \Big[& d\tau \wedge (k_1' g^1 \wedge g^2 + k_2' g^3 \wedge g^4) - \\ & - \frac{1}{2}(k_1 - k_2) g^5 \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \Big] , \end{aligned} \tag{3.28a}$$

$$\begin{aligned} F_3 = M \Big[& - g^5 \wedge \left\{ \frac{k_1'}{\tanh^2(\tau/2)} g^3 \wedge g^4 + k_2' \tanh^2\left(\frac{\tau}{2}\right) \wedge g^1 \wedge g^2 \right\} + \\ & + \frac{1}{2}(k_1 - k_2) d\tau \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \Big] , \end{aligned} \tag{3.28b}$$

where the prime denotes the differentiation with respect to τ, and we have used the relation

$$d(g^1 \wedge g^2) = -d(g^3 \wedge g^4) = -\frac{1}{2} g^5 \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) . \tag{3.29}$$

Now, utilising the relations

$$d(g^5 \wedge g^3 \wedge g^4) = d(g^5 \wedge g^1 \wedge g^2) = 0 , \tag{3.30}$$

$$d(g^1 \wedge g^3 + g^2 \wedge g^4) = g^5 \wedge (g^1 \wedge g^2 - g^3 \wedge g^4) , \tag{3.31}$$

the first of (2.30a) reduces to

$$\left(\frac{k_1'}{\tanh^2(\tau/2)} \right)' = \frac{k_1 - k_2}{2} , \qquad \left[k_2' \tanh^2\left(\frac{\tau}{2}\right) \right]' = -\frac{k_1 - k_2}{2} . \tag{3.32}$$

These equations are equivalent to

$$k_1' = \alpha(1 - F) \tanh^2\left(\frac{\tau}{2}\right) , \qquad k_2' = \alpha F \coth^2\left(\frac{\tau}{2}\right) , \tag{3.33}$$

where α is a constant and $F(\tau)$ obeys the differential equation

$$F'' = \frac{1}{2} \left[F \coth^2\left(\frac{\tau}{2}\right) + (F - 1) \tanh^2\left(\frac{\tau}{2}\right) \right] . \tag{3.34}$$

Since $(B_2 \cdot H_3)_Y$ is a constant multiple of $(k_1 k_1' + k_2 k_2') d\tau$ and h depends only on τ and x, the second equation of (2.30a) is automatically satisfied as in the previous cases. Hence, from the general arguments in section 2, for each solution to (3.34), we obtain a dynamical solution of the form $h = h_0(x) + h_1(y)$, where $h_0(x)$ is a linear function of x^μ and h_1 is a solution to (2.30b).

For example, we require that h is regular at $\tau = 0$ and approaches a constant at $\tau = \infty$, F and h are determined as [10]

$$F = \frac{\sinh\tau - \tau}{2\sinh\tau}\,, \tag{3.35a}$$

$$h(x,\tau) = h_0(x) + \alpha\frac{2^{2/3}}{4}\int_\tau^\infty du\,\frac{u\coth u - 1}{\sinh^2 u}\{\sinh(2u) - 2u\}^{1/3}\,. \tag{3.35b}$$

At large r, this solution behaves as

$$h \sim h_0(x) + \frac{81\alpha\sigma^4}{2r^4}\ln\frac{r}{\sigma}\,. \tag{3.36}$$

This behavior is the same as that of the conifold solution (3.12). From this we find that $\alpha\sigma^4$ corresponds to $g_s M^2$ representing the intensity of the G_3 flux. Hence, for a large G_3 flux, this solution provides a regular solution with a large warp factor. As discussed in section 3.1, this large warp factor stabilises the scale modulus for cosmological solutions with $h_0 = -p\,t + q$.

For reference, we gave an explicit expression for the general solution, which is in general singular at $r = 0$ or at $r = \infty$, in appendix B.

4. Supersymmetry breaking

In this section, we examine whether supersymmetry is preserved or not by the dynamical instability of the scale modulus. For simplicity, we only consider the case of no 3-form flux, $B_2 = C_2 = C_0 = \Phi = 0$ and assume that the ten-dimensional metric has the form

$$\begin{aligned} ds^2 &= g_{MN}dx^M dx^N \\ &= h^{-1/2}(x,r)ds^2(E^{3,1}) + h^{1/2}(x,r)[dr^2 + r^2 ds^2(\mathrm{Z}_5)]\,. \end{aligned} \tag{4.1}$$

4.1 Supersymmetry transformation

In the ten-dimensional type-IIB supergravity with $B_2 = C_2 = C_0 = \Phi = 0$, the local supersymmetry transformation of the spinor fields (gravitino ψ_M and dilatino λ) is given by [23, 24, 30 – 32]

$$\delta\lambda = 0\,, \tag{4.2a}$$

$$\delta\psi_M = \bar{\nabla}_M\epsilon\,, \tag{4.2b}$$

where ϵ is a ten-dimensional complex Weyl spinor satisfying the chirality condition $\Gamma^{11}\epsilon = \pm 1$, and the covariant derivative $\bar{\nabla}_M$ is given by

$$\bar{\nabla}_M = \nabla_M + \frac{i}{16\cdot 5!}\not{F}_5\,\Gamma_M \tag{4.3}$$

in terms of the ten-dimensional γ-matrices Γ^M satisfying

$$\Gamma^M\Gamma^N + \Gamma^N\Gamma^M = 2g^{MN}\,. \tag{4.4}$$

The bosonic fields are automatically invariant under local supersymmetric transformations because we are considering solutions with vanishing spinor fields.

For the metric (4.1), it is convenient to introduce $\gamma^\mu(\mu = 0, \cdots, 3)$, γ^r and $\gamma^l(l = 5, \cdots, 9)$ by

$$\Gamma^\mu = h^{1/4}\gamma^\mu\,, \qquad \Gamma^r = h^{-1/4}\gamma^r\,, \qquad \Gamma^l = \frac{1}{rh^{1/4}}\gamma^l\,. \tag{4.5}$$

Then, γ^μ give the SO(3, 1) γ-matrices, γ^l provide the γ-matrices of Z_5, and $(\gamma^r)^2 = 1$. We also define $\gamma_{(4)}$ and $\gamma_{(10)}$ by

$$\gamma_{(4)} = -i\,\gamma^0\,\gamma^1\,\gamma^2\,\gamma^3\,, \qquad \gamma_{(6)} = \gamma_{(4)}\Gamma^{11}\,, \tag{4.6}$$

so that $(\gamma_{(4)})^2 = (\gamma_{(6)})^2 = 1\,$.

In terms of these γ-matrices, the supersymmetry transformation in the background with the metric (4.1) is expressed as

$$\bar{\nabla}_\mu\,\epsilon = \left[{}^X\nabla_\mu + \frac{\partial_\nu h}{8h}\gamma^\nu{}_\mu - \frac{h'}{8h^{3/2}}\gamma_\mu\,\gamma^r\right]\epsilon\,, \tag{4.7a}$$

$$\bar{\nabla}_r\epsilon = \left[\partial_r + \frac{h'}{8h}\gamma_{(4)} - \frac{\partial_\mu h}{8h^{1/2}}\gamma^\mu\,\gamma^r\right]\epsilon\,, \tag{4.7b}$$

$$\bar{\nabla}_l\epsilon = \left[{}^Z\nabla_l + \frac{1}{2}\gamma_l\,\gamma^r - \frac{r\partial_\mu h}{8h^{1/2}}\gamma^\mu\,\gamma_l - \frac{rh'}{8h}\gamma^r\,\gamma_l\right]\epsilon\,, \tag{4.7c}$$

where the prime denotes differentiation with respect to r, and ${}^X\nabla_\mu$ and ${}^Z\nabla_l$ are the covariant derivatives with respect to the metrics, $ds^2(E^{3,1})$ and $ds^2(Y_5)$, respectively. The number of unbroken supersymmetries is determined by the number of covariantly constant (or Killing) spinor ϵ for which the right-hand sides of (4.7) vanish.

4.2 Consistency condition for the Killing spinor

Each generator ϵ of an unbroken supersymmetry should satisfy the consistency (integrability) condition

$$[\bar{\nabla}_M,\ \bar{\nabla}_N]\epsilon = 0\,. \tag{4.8}$$

By using the relation

$$\nabla_M\Gamma^N = 0\,, \qquad [\nabla_M,\ \nabla_N] = \frac{1}{4}R_{MNPQ}\Gamma^{PQ}\,, \tag{4.9}$$

the commutator of the covariant derivatives in the consistency condition (4.8) can be in general written

$$\begin{aligned}[\bar{\nabla}_M,\ \bar{\nabla}_N] &= \frac{1}{4}R_{MNPQ}\Gamma^{PQ} + \frac{2i}{16\cdot 5!}\nabla_{[M}\,\not{F}_5\,\Gamma_{N]} - \frac{1}{(16\cdot 5!)^2}[\not{F}_5\,\Gamma_M,\ \not{F}_5\,\Gamma_N]\\ &= \frac{1}{4}R_{MNAB}\Gamma^{AB} - \frac{i}{96}\nabla_{P_1}F_{P_2P_3P_4MN}\Gamma^{P_1\cdots P_4}(1\pm\Gamma^{11}) - \\ &\quad -\frac{1}{64\cdot 4!}F^{Q_1\cdots Q_4}{}_PF_{Q_1\cdots Q_4[M}\Gamma^P{}_{N]}(1\pm\Gamma^{11})\,. \end{aligned} \tag{4.10}$$

With the help of this consistency condition, let us examine how many supersymmetries exist. To begin with, for comparison, we recall the results for the well-studied case of the static background [30, 32–34]. First, for the case in which $h = \text{const}$ or $h = C/r^4$, the only non-trivial consistency condition is given by

$$[\bar{\nabla}_l,\ \bar{\nabla}_m]\epsilon = \frac{1}{4}C_{lmpq}(\mathrm{Z}_5)\gamma^{pq}\epsilon\,, \tag{4.11}$$

where C_{lmpq} is the Weyl tensor of the Z_5 space. Hence, the number of supersymmetries is determined by the number of solutions to the spinor equation

$$\bar{\nabla}_l\epsilon_0 = \left({}^{\mathrm{Y}_5}\nabla_l \pm \frac{1}{2}\gamma_l\right)\epsilon_0 = 0\,. \tag{4.12}$$

In particular, for the ten-dimensional Minkowski spacetime [24] and for $\mathrm{AdS}^5\times\mathrm{S}^5$ [34], the background has the full supersymmetry.

Next, we consider the static conifold background with $h = h_0 + C/r^4(h_0C \neq 0)$ [30]. For this background, the $[\mu, r]$-component of the consistency condition reads

$$0 = [\bar{\nabla}_\mu,\ \bar{\nabla}_r]\epsilon = 2h^{-1/4}(h^{-1/4})''\,\gamma_\mu\gamma^r\,\left(\gamma_{(4)} - 1\right)\epsilon\,. \tag{4.13}$$

Hence, ϵ should satisfy

$$\left(\gamma_{(4)} - 1\right)\epsilon = 0\,. \tag{4.14}$$

We can show that this condition and (4.11) are the only non-trivial consistency conditions. Hence, one half of the supersymmetries in the previous case are broken in this conifold background [30, 33].

Now, let us consider the background with $\partial_\mu h \neq 0$: In this case, from the $[\mu,\nu]$-components of the consistency condition, we obtain

$$0 = b^\mu c^\nu[\bar{\nabla}_\mu,\ \bar{\nabla}_\nu]\epsilon = -\frac{(Dh_0)^2}{32h^2}b_\mu\gamma^\mu c_\nu\gamma^\nu\epsilon\,, \tag{4.15}$$

where b_μ and c_ν are linearly independent vectors satisfying the conditions $b^\mu\partial_\mu h = c^\mu\partial_\mu h = 0$. From this, it follows that if $a_\mu = \partial_\nu h_0$ is not a null vector, there exists no non-trivial solution to the consistency condition, and the supersymmetry is completely broken. In contrast, when a_μ is a null vector, we find that the consistency condition is equivalent to

$$\gamma_{(4)}\epsilon = \epsilon\,, \qquad (D_\mu h)\gamma^\mu\epsilon = 0\,, \qquad \frac{1}{2}C_{lmpq}(\mathrm{Y}_5)\gamma^{pq}\epsilon = 0\,. \tag{4.16}$$

Hence, a quarter of the supersymmetries in the case of $h_0C = 0$ are preserved.

Finally, we comment on the degree of supersymmetry breaking for the dynamical background. One natural measure for that is obtained from (4.15). It is the mass scale corresponding to $(Dh_0)^2/h^2$. If we consider the induced effective mass for the spinor field, we obtain a similar mass scale. This mass scale diverges at the naked singularity where h vanishes. Hence, for the cosmological situation $h = -p\,t + q$ argued in the previous section, the degree of supersymmetry breaking increases as the universe approaches the big-lip singularity. In contrast, in the region with a large warp factor, the supersymmetry breaking becomes negligible.

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5. Discussion

In the present paper, we have studied the dynamical stability in the moduli sector of supersymmetric solutions for the conifold-type warped compactification of the ten-dimensional type-IIB supergravity, by looking for extensions of supersymmetric solutions to those that depends on the four-dimensional coordinates. We have found that for many of the well-know solutions compactified on a conifold, resolved conifold or deformed conifold, such extensions exist and exhibit a dynamical instability. Further, this instability is associated with supersymmetry breaking. This feature is expected to be shared by a quite wide class of supersymmetric solutions beyond the examples considered in the present paper, because the result has been obtained by analysing the general structure of solutions for warped compactification with flux of the type-IIB supergravity under ansatz that is natural to include supersymmetric solutions as a special case.

The dynamical solutions found in the present paper can be always obtained by replacing the constant modulus h_0 in the warp factor $h = h_0 + h_1(y)$ for supersymmetric solutions by a linear function $h_0(x)$ of the four-dimensional coordinates x^μ. Since h_0 corresponds to the scaling degree of freedom of the internal space, this implies that the dynamical instability occurs in the Kähler modulus representing the scale of the internal space. This type of instability in the moduli sector itself is not surprising, because constant moduli have flat potentials in effective four-dimensional theories. In fact, in the large r region in which h becomes independent of r, it is expected that the behavior of the solution is well described by an effective four-dimensional theory. Hence, the instability found in the present paper corresponds to a decompactifying run-away solution in an effective theory. However, it is not expected generally in effective theories that such instabilities give rise to significant position-sensitive time dependence in the structures of the four-dimensional spacetime and of the internal space. It is because most effective theories do not take account of the warped structure [8, 11]. In fact, we have found that the degree of instability significantly depends on the position in the internal space. In the conifold-type examples considered in the present paper, the instability is most enhanced at infinity, while near the conifold singularity or in the region with a large warp factor for the deformed-conifold case, the instability is strongly suppressed. Thus, the moduli stability is closely connected with the large warp factor, which has been used to resolve the hierarchy problem in the context of the flux compactification [11]. This feature may also play an important role in constructing realistic universe model in the KKLT scheme or in the braneworld scheme.

In this connection, we would like to comment on some subtle points. First, the instability we have found is a global mode on an open internal space. Hence, one may suspect that such an instability does not really occur in a model with a compact internal space. In particular, in a model in which the scale modulus is stabilised by quantum effects in the effective four-dimensional theory, the instability may be able to grow only when the effective kinetic energy $\dot{h}^2/h^2$ of the modulus exceeds the height of the potential barrier, taking account of the correspondence between the ten-dimensional theory and the effective theory mentioned above. However, it is quite difficult to make clear this point by an explicit analysis because there exists no warped ten-dimensional model that takes account of quantum effects and their backreaction on the geometry. Actually, there exists at present no warped ten-dimensional model with smooth compact internal space in which the back-reactions of flux and negative charges of orientifold planes are properly taken into account, because such negative charges produce naked singularities. Apart from this global problem, there is also a possibility that a similar instability occurs locally. Such a local instability may grow even in a model with quantum moduli stabilisation if the spatial scales of the instability are smaller than the length scale corresponding to the stabilisation energy scale. It will be interesting to see whether such a local instability exists by a linear perturbation analysis (cf. ref. [35]).

Another subtle point is that we have found instability only in the scale modulus. For example, in the Gibbons-Lü-Pope solution, the warp factor h can have a large number of constant moduli corresponding to the positions of $D3$ branes in addition to the scale modulus. Such a solution is contained in the class of solutions analysed in the present paper, but no instability has been found in these additional moduli. One possible reason for this is that the ansatz concerning the structure of the ten-dimensional metric is too restrictive. A linear perturbation analysis may also be useful in clarifying this point.

Finally, we would like to point out that the degree of supersymmetry breaking is also closely related to the warp factor, which can be interpreted as the cosmic scale factor in the cosmological context. Hence, the cosmic expansion, the hierarchy and the supersymmetry breaking are tightly connected. Although the examples considered in the present paper do not provide realistic cosmological models, this feature may be utilised to solve the hierarchy problem and the supersymmetry breaking problem in a realistic higher-dimensional cosmological model.

Acknowledgments

The authors would like to thank Shigeki Sugimoto for valuable discussions. This work is supported by the JSPS grant No. 15540267 (H. K.) and by the Yukawa fellowship (K. U.).

A. Solutions for h_0 and X_4

In this appendix, we find a general solution for h_0 and $ds^2(\mathrm{X}_4)$ to (2.30c) and (2.32a).

First, we consider the case in which $D_\mu h_0 \neq 0$ and $(Dh_0)^2 \neq 0$. In this case, in terms of the synchronous coordinates with respect $h_0 = t$, $ds^2(\mathrm{X}_4)$ can be written

$$ds^2 = N(t,z)dt^2 + q_{ij}(t,z)dz^i dz^j \,. \tag{A.1}$$

In this coordinate system, the equation

$$D_\mu D_\nu h_0 = -\Gamma^t_{\mu\nu} = \lambda g_{\mu\nu} \tag{A.2}$$

is equivalent to

$$N^{-1}\partial_t N = -2\lambda N \,, \qquad \partial_i N = 0 \,, \qquad \frac{1}{2N}\partial_t q_{ij} = \lambda \,. \tag{A.3}$$

A general solution to this equation is

$$N = \frac{1}{2\lambda t + c}, \qquad q_{ij} = |2\lambda t + c| q^0_{ij}(z). \tag{A.4}$$

In terms of τ defined by

$$|2\lambda t + c| = c_1 e^{2\lambda\tau}, \tag{A.5}$$

this metric can be put into the form

$$ds^2 = c_1^2 e^{2\lambda\tau}\left[\pm d\tau^2 + q^0_{ij}(z) dz^i dz^j\right], \tag{A.6}$$

where $\pm$ is the sign of $2\lambda t + c$. For this metric, the condition $R_{\mu\nu}(\mathrm{X}_4) = 0$ is equivalent to

$$R_{ij}(q^0) = \pm 2\lambda^2 q^0_{ij}. \tag{A.7}$$

We can show by direct calculations that $ds^2(\mathrm{X}_4)$ is flat under this condition.

Next, we consider the case in which $Dh_0 \neq 0$ and $(Dh_0)^2 = 0$. In this case, from $D_\mu(Dh_0)^2 = 2D^\nu h_0 D_\mu D_\nu h_0 = 0$ it follows that λ should vanish. Further, $u = h_0$ is a null coordinate and Du becomes a null Killing. Therefore, the metric can be written

$$ds^2 = 2du\left(d\rho + a du + b_i dz^i\right) + q_{ij} dz^i dz^j, \tag{A.8}$$

where b_i and q_{ij} are functions independent of ρ. If we define the null vectors $e_\pm$ as

$$e_+ = \partial_u - a\partial_\rho, \qquad e_- = \partial_\rho, \tag{A.9}$$

the Ricci curvature is expressed as

$$R_{-\mu} = 0, \tag{A.10a}$$
$$R_{++} = -\triangle a + D^i(\partial_u b_i) - \partial_u K - K_{ij}K^{ij} + 2B^2, \tag{A.10b}$$
$$R_{+i} = -\partial_i K + D_j K_i^j + \epsilon_{ij} D^j B, \tag{A.10c}$$
$$R_{ij} = R_{ij}(q), \tag{A.10d}$$

where

$$K_{ij} = \frac{1}{2}\partial_u q_{ij}, \qquad B = \frac{1}{2}\epsilon^{ij}\partial_{[i} b_{j]}. \tag{A.11}$$

First, from the condition $R_{ij}(q) = 0$, we can set $q_{ij} = \delta_{ij}$ by an appropriate coordinate transformation. In this coordinate system, $K_{ij} = 0$. Hence from $R_{+i} = 0$ we have $B = B(u)$. This implies that b_i can be written

$$b_i = -B\epsilon_{ij} z^j + \partial_i C(u, z). \tag{A.12}$$

Inserting this to $R_{++} = 0$, we obtain

$$\triangle_2(-a + \partial_u C) + 2B^2 = 0, \tag{A.13}$$

whose general solution is

$$a = \partial_u C + \frac{B^2}{2} z^i z_i + \frac{f}{2}, \tag{A.14}$$

where f is an arbitrary solution to $\triangle_2 f = 0$. Thus, after the coordinate transformation $\rho + C \to \rho$, the metric can be expressed as

$$ds^2 = 2du\, d\rho + f du^2 + (dz^1 - Bz^2 du)^2 + (dz^2 + Bz^1 du)^2 \,. \tag{A.15}$$

The only non-vanishing components of the curvature tensor for this metric are

$$R_{uiuj} = -\frac{1}{2}\partial_i \partial_j f \,. \tag{A.16}$$

Therefore, if f is linear with respect to z^i, the spacetime is flat. For example, in terms of the coordinates defined by

$$t = \frac{1}{2}u - \rho\,, \qquad z = -\frac{1}{2}u - \rho\,, \tag{A.17a}$$

$$x = z^1 \cos\beta + z^2 \sin\beta\,, \qquad y = -z^1 \sin\beta + z^2 \cos\beta\,, \tag{A.17b}$$

where $\beta = \int B du$, the metric can be written

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + f(x, y, t - z)(dt - dz)^2\,, \tag{A.18}$$

where $f(x, y, t - z)$ is a harmonic function with respect to x and y.

B. General solution for the deformed conifold

In this appendix, we give the general solution for the deformed conifold case discussed in section 3.3.

First, the general solution for F , k_1 and k_2 are given by

$$F = \frac{1}{2}\left(1 - \frac{\tau}{\sinh\tau}\right) + \frac{C_1}{\sinh\tau} + C_2 \frac{2\tau - \sinh(2\tau)}{2\sinh(\tau)}\,, \tag{B.1a}$$

$$\begin{aligned} k_1 &= C_0 + \alpha\frac{1 - e^{2\tau} + \tau(1 + e^{2\tau})}{2(1 + e^{\tau})^2} + \alpha C_1 \frac{2e^{\tau}}{(1 + e^{\tau})^2} + \\ &\quad + \alpha C_2 \left\{ \frac{e^{3\tau} + e^{2\tau} - 5e^{\tau} - 1}{2e^{\tau}(1 + e^{\tau})} - 2\tau \frac{e^{2\tau} + e^{\tau} + 1}{(1 + e^{\tau})^2} \right\}, \end{aligned} \tag{B.1b}$$

$$\begin{aligned} k_2 &= C_0 + \alpha\frac{1 - e^{2\tau} + \tau(1 + e^{2\tau})}{2(1 - e^{\tau})^2} - \alpha C_1 \frac{2e^{\tau}}{(1 - e^{\tau})^2} - \\ &\quad - \alpha C_2 \left\{ \frac{e^{3\tau} - e^{2\tau} - 5e^{\tau} + 1}{2e^{\tau}(e^{\tau} - 1)} + 2\tau \frac{e^{2\tau} - e^{\tau} + 1}{(e^{\tau} - 1)^2} \right\}. \end{aligned} \tag{B.1c}$$

For the deformed conifold, (2.30b) reads

$$\frac{1}{\sinh^2\tau}\partial_\tau \left(\frac{6K^2 \sinh^2\tau}{\sigma^3}\partial_\tau h \right) = -g_s^{-1}(H_3 \cdot H_3)_Y\,, \tag{B.2}$$

where $(H_3 \cdot H_3)_Y$ in the right-hand side is given by

$$H_3 \cdot H_3 = \frac{24 g_s^2 \alpha^2}{\sigma^9}\left[\frac{(1 - F)^2}{\cosh^4(\tau/2)} + \frac{F^2}{\sinh^4(\tau/2)} + \frac{2(k_1 - k_2)^2}{\alpha^2 \sinh^2\tau} \right]. \tag{B.3}$$

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Hence, the general solution for the warp factor h can be obtained by integrating

$$
\begin{aligned}
\partial_\tau h = & -\frac{16 \cdot 2^{2/3} g_s \alpha^2}{\sigma^6 (\sinh(2\tau) - 2\tau)^{2/3} \sinh^3 \tau} \times \\
& \times \Big[-\frac{1}{4} (\sinh(2\tau) - 2\tau)(\tau \cosh \tau - \sinh \tau) + 2C_1 (\tau \cosh \tau - \sinh \tau) + \\
& + C_2 \Big\{ 2\tau^2 \cosh \tau - \frac{\tau}{2} (\sinh(3\tau) + 5 \sinh \tau) + e^{-\tau} \sinh \tau \Big\} - \\
& - 4C_1 C_2 (\tau \cosh \tau - \sinh \tau) - 8C_1^2 \cosh \tau \\
& + C_2^2 \Big\{ -4\tau^2 e^{2\tau} \cosh \tau + 4\tau e^{2\tau} \sinh \tau + \\
& \qquad + \frac{1}{8} \left(e^{7\tau} - 3e^{5\tau} - 6e^{3\tau} + 18e^{\tau} - 11e^{-\tau} + e^{-3\tau} \right) \Big\} \\
& + C_3 \sinh^3 \tau \Big] .
\end{aligned}
\tag{B.4}
$$

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